3

Math in Economics Series

Calculating Areas on Demand & Supply Graphs

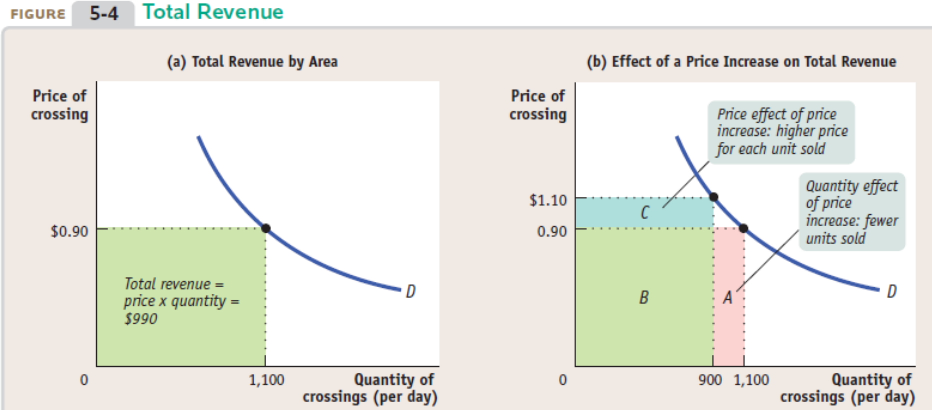
This tutorial covers how to calculate the area of a:

- rectangle
- right triangle
- non-right triangle
- trapezoid

1 Rectangles

The left panel of Krugman, Wells, and Graddy's (2014) Figure 5–4 contains a rectangle that's shaded green. The area of that rectangle is economically important because it equals the tax revenue the government collects from a 90-cent tax to cross a bridge. Let's see why.

FIGURE 5-4 Total Revenue



The green rectangle in panel (a) represents total revenue generated from 1,100 drivers who each pay a toll of $0.90. Panel (b) shows how total revenue is affected when the price increases from $0.90 to $1.10. Due to the quantity effect, total revenue falls by area *A*. Due to the price effect, total revenue increases by the area *C*. In general, the overall effect can go either way, depending on the price elasticity of demand.

The base of the shaded rectangle is 1,100 crossings per day. The height of the rectangle is 90 cents per crossing. Since the area of a rectangle is base times height, the area of the shaded rectangle is

$$1{,}100 \times 0.90 = 990$$

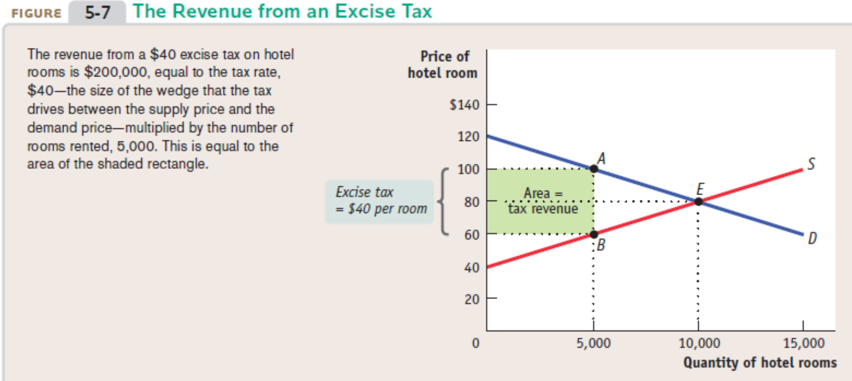
dollars per day.

The final step is to connect the area of the rectangle to tax revenue. That's easy because tax revenue is the tax rate (e.g., 90 cents per crossing) times the quantity taxed (e.g., 1,100 crossings per day), which is exactly what we calculated above.

Practice Question

In Krugman et al's (2014) Figure 5–7, how much tax revenue does the government collect if the tax rate is $40 per room? That is, what is the area of the shaded rectangle?

FIGURE 5-7 The Revenue from an Excise Tax

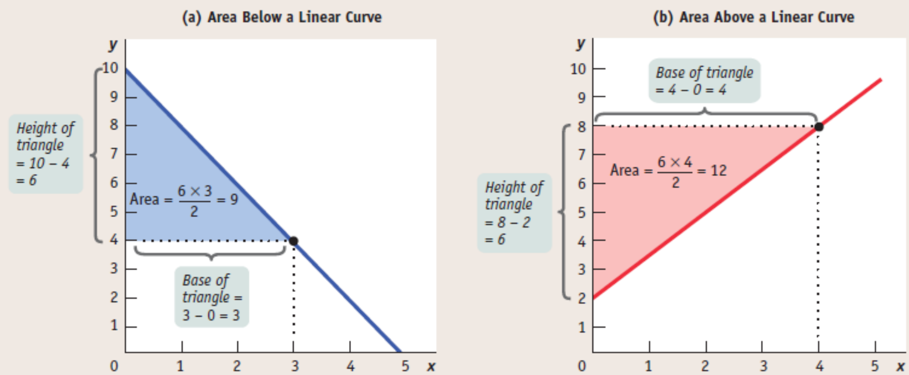


2 Right Triangles

Each panel of Krugman, Wells, and Graddy's (2014) Figure 2A–7 contains a shaded right triangle. Each triangle is a right triangle because two of its sides meet to form a right (i.e., 90 degree) angle.

The area of each triangle has important meaning in economics, so economics students learn to calculate the areas of these (and other) triangles. To calculate the area of a right triangle, we multiple the height by the base and divide by 2. That is, the area of a right triangle is half the area of a rectangle with the same base and height. One of the two sides that make a right angle is the height of the triangle and the other is its base.

FIGURE 2A-7 Calculating the Area Below and Above a Linear Curve



The area above or below a linear curve forms a right triangle. The area of a right triangle is calculated by multiplying the height of the triangle by the base of the triangle, and dividing the result by 2. In panel (a) the area of the shaded triangle is $\frac{6 \times 3}{2} = 9$. In panel (b) the area of the shaded triangle is $\frac{6 \times 4}{2} = 12$.

The area of the right triangle in panel (a) is

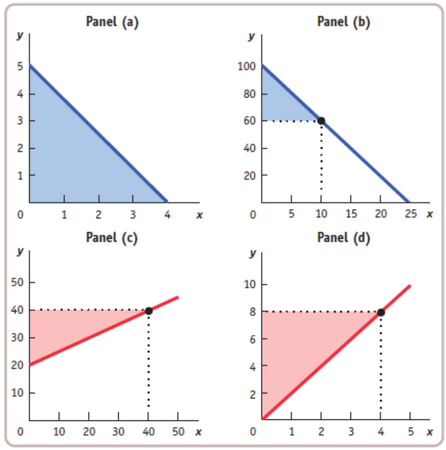
$$\frac{(10 - 4) \times (3 - 0)}{2} = \frac{6 \times 3}{2} = 9.$$

The area of the right triangle in panel (b) is

$$\frac{(8 - 2) \times (4 - 0)}{2} = \frac{6 \times 4}{2} = 12.$$

Practice Question

Using this figure from Krugman, Wells, and Graddy (2014, Appendix to Chapter 2), calculate the area of the shaded right triangle in each panel.

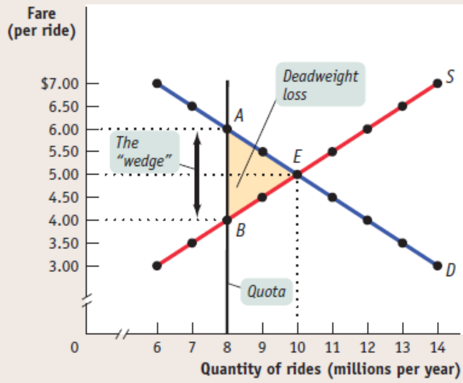


3 Non-Right Triangles

The "base $\times$ height $\div$ 2" rule applies to triangles without right angles, too. We just have to be careful how we measure base and height.

Let's compute the area of $\triangle ABE$, the triangle defined by points A, B, and E in Krugman, Wells, and Graddy's (2014) Figure 4–14. Since this triangle has one vertical side (AB), its height is clear enough; it's the distance from A to B. But there's no obvious base in $\triangle ABC$. In this case, the relevant base is the length of the line segment from point E that's perpendicular to line segment AB. That mouthful is simply the answer to the question, "How wide is the triangle?"

FIGURE 4-14 Effect of a Quota on the Market for Taxi Rides



Fare (per ride)	Quantity of rides (millions per year)	
	Quantity demanded	Quantity supplied
$7.00	6	14
6.50	7	13
6.00	8	12
5.50	9	11
5.00	10	10
4.50	11	9
4.00	12	8
3.50	13	7
3.00	14	6

The table shows the demand price and the supply price corresponding to each quantity: the price at which that quantity would be demanded and supplied, respectively. The city government imposes a quota of 8 million rides by selling licenses for only 8 million rides, represented by the black vertical line. The price paid by consumers rises to $6 per ride, the demand price of 8 million rides, shown by point *A*. The supply price of 8 million rides is only $4 per ride, shown by point *B*. The difference between these two prices is the quota rent per ride, the earnings that accrue to the owner of a license. The quota rent drives a wedge between the demand price and the supply price. And since the quota discourages mutually beneficial transactions, it creates a deadweight loss equal to the shaded triangle.

So our triangle $\triangle ABC$ has height AB and a base that equals 2 million rides per year. The area of $\triangle ABE$ is

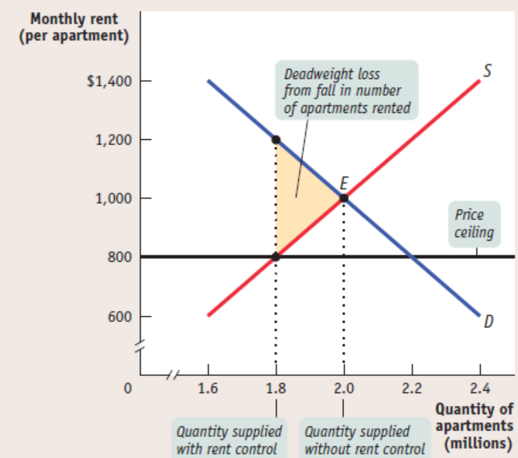
$$\frac{(10-8)\times(6-4)}{2} = \frac{2\times 2}{2} = 2.$$

Practice Question

What is the area of the shaded triangle in Krugman, Wells, and Graddy's (2014) Figure 4–8?

FIGURE 4-8 A Price Ceiling Causes Inefficiently Low Quantity

A price ceiling reduces the quantity supplied below the market equilibrium quantity, leading to a deadweight loss. The area of the shaded triangle corresponds to the amount of total surplus lost due to the inefficiently low quantity transacted.



4 Trapazoids

Rectangles and triangles are the most common geometric objects on graphs in economics courses. Trapezoids also pop up. For instance, in Krugman, Wells, and Graddy's (2014) Figure 5–7, combining the shaded-green rectangle and the triangle ABC produces a trapezoid. To compute the area of the trapezoid, we simply add the area of the triangle to the area of the rectangle.

References

Krugman, Paul, Wells, Robin, and Graddy, Kathryn. *Essentials of Economics*. Worth Publishers, 2014.

Additional Resources

For questions about this topic, see a **Algebra** or **Math Ed** tutor at the Dolciani Mathematics Learning Center (Hunter East, 7th floor) or any tutor in the Economics Tutoring Center (Hunter West, 15th floor).

The Dolciani Mathematics Learning Center also provides related tutorials on several platforms—

CDs, DVDs, and online. Online access is through PLATO. Visit the front desk at the Math Learning Center to create a PLATO account.

Resources at the Dolciani Mathematics Learning Center

Topic	*Situational DVDs*	*Tutorial CDs/DVDs*	PLATO
right triangle	J3	Y10, Z5	Support for Triangles: Area, Perimeter, & Pythagorean Theorem
non-right triangle		Y9, Y10, Z6	Support for Triangles: Area, Perimeter, & Pythagorean Theorem

Acknowledgements

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