4 Calculating Growth Rates, Inflation Rates, and Elasticities

This tutorial covers **how to** calculate the:

- percentage change
- growth rate of real GDP from one period to the next
- inflation rate
- percentage changes from one point to another point on a demand curve
- price elasticity of demand using the midpoint method

1 Percentage Change

Good news! Today you learn that you're getting a big raise. Your hourly wage rate is jumping from \$10 to \$14. That's \$4 more per hour. It's also a 40 percent raise. That is, your new hourly wage is 40 percent greater than your current hourly wage.

Just to be extra clear, the 40-percent result comes from expressing the \$4 raise relative to a base value for your hourly wage. The convention is to use your current wage (\$10) as the base value. So your raise in percentage terms is

$$\frac{14-10}{10} = \frac{4}{10} = .40 = 40\%$$

Bad news! At the end of the pay period, the geniuses in payroll discover that your raise was a mistake. Sorry, but your hourly wage is falling back to \$10. Let's express your pain in percentage terms. Your wage falls from \$14 to \$10; to express the \$4 cut in pay in percentage terms, we divide by starting wage, which is \$14 this time.

$$\frac{10-14}{14} = \frac{-4}{14} = -.286 = -28.6\%$$

You've ended up right back where you started because a \$4 raise followed by a \$4 pay cut brings you back to your \$10 wage. But those two changes amount to a 40% increase in pay followed by a 28.6% decrease in pay. In percentage terms, you appear to be 11.4% ahead, although you're clearly not. That's annoying.

As this example demonstrates, percentage changes are sensitive to where we move *from*. The following calculations of growth rates and inflation rates use this conventional method to compute the percentage changes. The final section explains how, in the context of elasticities, the midpoint method for calculating percentage changes eliminates the sensitivity to starting points.

2 Growth Rates

A country's **real growth rate** is, in terms of calculation, like the rate of growth of your hourly wage. We simply compute the percentage change in the country's real GDP from one period to the next.

If a country's real GDP was $9 trillion in 2018 and $9.6 trillion in 2019, then its real growth rate is

$$\frac{\$9.6\text{ trillion} - \$9\text{ trillion}}{\$9\text{ trillion}} = .0667 = 6.67\%$$

per year from 2018 to 2019.

Sometimes we calculate the annual growth rate over several years. If the country's real GDP was $7.14 trillion in 2009, then real output grew 34.45% over the 10 years from 2009 to 2019.

$$\frac{\$9.6\text{ trillion}}{\$7.14\text{ trillion}} = 1.3445$$

which means that real GDP in 2019 was as big as in 2009 *plus* .3445 times 2009's real output. That is, real GDP grew 34.45% over those 10 years.

We can express this real growth rate in annual terms. The annual growth rate is the answer to the question: If real GDP grew at a constant rate from year to year, what would that annual real growth rate have to be to deliver the 34.45% growth over 10 years?

To find the annual real growth rate, we take the nth root of 1.3445; in this case, $n = 10$.

$$1.3445^{\frac{1}{10}} = 1.03$$

So if we multiplied 1.03 by itself 10 times (i.e., 1.03^{10}), we'd end up with 1.3445. And that means that real GDP grew by 3% per year over the 10 years from 2009 to 2019.

This doesn't mean that real GDP grew by 3.0% each year from 2009 to 2019. It just means that the actually growth rates over those 10 years deliver the same eventual growth as 3% growth every year.

Now for the formula. The annual growth rate of real GDP over n years is

$$\left[\left(\frac{\text{real GDP}_{t+n}}{\text{real GDP}_t}\right)^{1/n} - 1\right]$$

where t is the starting year (e.g., 2009) and $t + n$ is the ending year (e.g., 2019).

Practice Question

This question comes from problem 21 in Cowan and Tabarrok (2018, Chapter 26).

In the United States, real GDP was \$323 billion in 1929 and \$206 billion in 1933. In percentage terms, how much *smaller* was real GDP in 1933? What was the annual real growth rate over the four years from 1929 to 1933? [Hint: *growth* rates can be negative.]

3 Inflation Rates

The **inflation rate** is the percentage change in the level of prices, which we measure with the Consumer Price Index (CPI). We'll work with the CPI measured relative to prices in the early 1980s when the value of the CPI was 100. The CPI in 2017 was 245.120; in 2018, it was 251.107.

From 2017 to 2018, the percentage change in the CPI was 2.4%.

$$\frac{251.107 - 245.120}{245.120} = .0244 = 2.44\%.$$

That's the annual rate of inflation from 2017 to 2018.

The CPI was 172.1 in 2000. Thus the price level in 2018 was 1.459 (i.e., 251.107/172.1) times the price level at the turn of the century. We could also say that prices grew 45.9% over these 18 years.

To express the 45.9% rise in prices as an *annual* inflation rate, take the 18th root of 1.459 and subtract 1.

$$1.459^{\frac{1}{18}} = 1.0212$$

So the annual inflation rate over these 18 years was 2.12%.

Practice Question

The values of the CPI in 2010 and 2011 were 218.056 and 224.939, respectively. What was the rate of inflation from 2010 to 2011?

What was the annual inflation rate over the 8 years from 2010 to 2018?

4 Elasticities

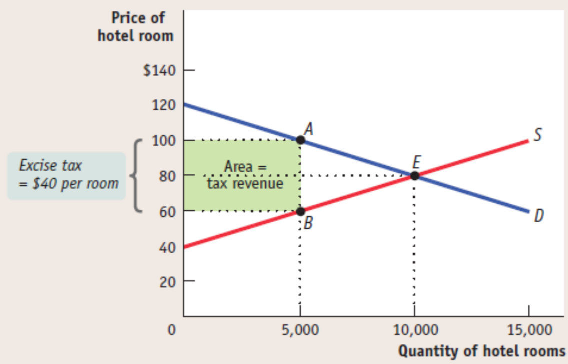
The **price elasticity of demand** is the percentage change in quantity demanded of a good divided by the percentage change in its price. Since price and quantity demanded move in opposite directions along a demand curve, the price elasticity of demand of a good is negative. Some textbooks, however, present the absolute value of the price elasticity of demand.

In Krugman, Wells, and Graddy's (2014) Figure 5–7, A and E are two points on the demand curve for hotel rooms. If the price is \$100 per room, people rent 5,000 hotel rooms per night;

that's point A. They get 10,000 rooms if the price per room is \$80, which is point E.

FIGURE 5-7 The Revenue from an Excise Tax

The revenue from a \$40 excise tax on hotel rooms is \$200,000, equal to the tax rate, \$40—the size of the wedge that the tax drives between the supply price and the demand price—multiplied by the number of rooms rented, 5,000. This is equal to the area of the shaded rectangle.



From A to E. Let's compute the price elasticity of demand associated with the fall in price from \$100 (the starting price) to \$80 (the ending price) per room. The percentage change in price is

$$\frac{\$80 - \$100}{\$100} = \frac{-\$20}{\$100} = -0.2 = 20\%$$

And the percentage change in quantity demanded is

$$\frac{10{,}000 - 5{,}000}{5{,}000} = \frac{5{,}000}{5,000} = 1.0 = 100\%$$

So people respond to the 20% fall in the price of hotel rooms by raising the number of hotel rooms they demand by 100%. Notice that we express these changes in percentage terms by dividing by the starting values—\$100 for the price and 5,000 for the quantity.

The price elasticity of demand for hotel rooms is the ratio of the percentage change in quantity demanded to the percentage change in price. From A to E,

$$\text{price elasticity of demand} = \frac{100\%}{-20\%} = -5$$

Now let's work in the other direction.

From E to A. If the price per room rises from \$80 (the starting price) to \$100 (the ending price), the percentage changes in price and quantity demanded are

$$\frac{\$100 - \$80}{\$80} = \frac{\$20}{\$80} = 0.25 = 25\%$$

$$\frac{5{,}000 - 10{,}000}{10{,}000} = \frac{-5{,}000}{10,000} = -0.5 = -50\%$$

In this direction, people respond to the 25% rise in the price of hotel rooms by lowering the number of hotel rooms they demand by 50%. In addition to changing signs (since were raising rather than lowering price), the magnitudes change with the change in the direction of the change because the starting values in the denominator change to the smaller price and larger quantity.

From E to A, the price elasticity of demand is

$$\text{price elasticity of demand} = \frac{-50\%}{25\%} = -2$$

which is much smaller (in absolute value) than the price elasticity we computed in the opposite direction. The price elasticity of demand is clearly sensitive to the direction of the change or where we move **from**.

The **midpoint method** eliminates the sensitivity (or multiplicity) of the elasticity calculation to the direction of the change. Rather than specify the direction of the change (e.g., from A to E), we specify the range of the change. Over the range of prices from \$80 to \$100 per room, the price elasticity of demand is -3.

$$\frac{\frac{10{,}000-5{,}000}{7{,}500}}{\frac{80-100}{90}} = \frac{66.67\%}{-22.22\%} = -3$$

Notice that the denominator in each percentage-change expression is its midpoint value—7,500 rooms (halfway between 5,000 and 10,000) and \$90 per room (halfway between \$80 and \$90).

Practice Question

The price of milk per gallon rises from \$2.85 to \$3.15, and dairy farmers increase the quantity supplied of milk from 9,000 to 11,000 gallons per month. What is the price elasticity of supply of milk using the midpoint formula?

References

Cowen, Tyler, and Tabarrok, Alex. *Modern Principles of Economics*, 4/e. Worth Publishers, 2018.

Krugman, Paul, Wells, Robin, and Graddy, Kathryn. *Essentials of Economics*. Worth Publishers, 2014.

Additional Resources

For questions about this topic, see an **Algebra** tutor at the Dolciani Mathematics Learning Center (Hunter East, 7th floor) or any tutor in the Economics Tutoring Center (Hunter West, 15th floor).

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Topic	*Situational DVDs*	*Tutorial CDs/DVDs*	PLATO
operations on fractions		A1, A3, A4, S3	Support for Fractions
percents		D4, X3	Support for Percents
converting fractions, decimals, percents		D4, X3	Support for Decimals

Acknowledgements

Shaoying Ma created this and other tutorials in the *Math for Economics* series. Kenneth McLaughlin supervised her work and edited the final product.